Sample Pages

Algebra 2

Lesson 12 **Quadratic Formula**

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In Algebra 2, students master factoring polynomials, quadratic formula, graphing conic sections and other topics.

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AUSTRALIAN EDITION

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Quadratic Formula

A ***quadratic*** is an equation that has an unknown or variable raised to the second power, as in Y^2 or A^2. In factoring and in completing the square, we have been dealing exclusively with quadratic equations. So far, we can find the solution to a quadratic equation by factoring it, or if this fails, by completing the square. In this lesson we are going to complete the square with variables in order to discover a formula to solve all quadratics. If you've mastered the previous lesson, try solving the following equation by completing the square, and then compare your solution with mine.

$$AX^2 + BX + C = 0$$

Divide by the coefficient of X^2.

$$\frac{AX^2}{A} + \frac{BX}{A} + \frac{C}{A} = 0$$

$$X^2 + \frac{BX}{A} + \frac{C}{A} = 0$$

Add the opposite of the third term to both sides.

$$X^2 + \frac{BX}{A} \qquad = -\frac{C}{A}$$

Take one-half of the coefficient of the middle term, square it, and add the result to both sides.

$$X^2 + \frac{BX}{A} + \left(\frac{B}{2A}\right)^2 = -\frac{C}{A} + \left(\frac{B}{2A}\right)^2$$

Factor the left side.

$$(X + \frac{B}{2A})^2 = -\frac{C}{A} + \frac{B^2}{4A^2}$$

Combine terms on the right.

$$(X + \frac{B}{2A})^2 = -\frac{4AC}{4A^2} + \frac{B^2}{4A^2}$$

Take the square root of both sides.

$$X + \frac{B}{2A} = \sqrt{-\frac{4AC}{4A^2} + \frac{B^2}{4A^2}} = \pm\frac{\sqrt{-4AC + B^2}}{2A}$$

Subtract B/2A from both sides, and combine.

$$X = -\frac{B}{2A} \pm \frac{\sqrt{-4AC + B^2}}{2A}$$

The ***quadratic formula***! This is the form in which it is usually written.

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Example 1

Let's try an equation that we can answer by factoring, and "plug in" the values for A, B, and C. Remember that to find A, B, and C, the equation must be in the form $AX^2 + BX + C = 0$.

$$X^2 + 5X + 6 = 0$$

A = 1, B = 5, and C = 6

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$X = \frac{-5 \pm \sqrt{5^2 - 4 \cdot 1 \cdot 6}}{2 \cdot 1}$$

$$X = \frac{-5 \pm \sqrt{25-24}}{2} = \frac{-5 \pm \sqrt{1}}{2}$$

$$X = \frac{-5 \pm 1}{2} = \frac{-4}{2} \text{ or } \frac{-6}{2} = -2 \text{ or } -3$$

We can also solve $X^2 + 5X + 6 = 0$ by factoring.

$$X^2 + 5X + 6 = 0$$
$$(X+2)(X+3) = 0$$
$$X + 2 = 0 \quad X + 3 = 0$$
$$X = -2 \quad X = -3$$

For this problem, it would have much easier to solve by factoring. Try factoring first, and if it doesn't work, use the quadratic formula. Here is another problem to try.

Example 2

Find the factors of $2X^2 = -7X - 4$.

To find A, B, and C, the equation must be in the form $AX^2 + BX + c = 0$.

$$2X^2 + 7X + 4 = 0$$

A = 2, B = 7, and C = 4

$$X = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$X = \frac{-7 \pm \sqrt{7^2 - 4 \cdot 2 \cdot 4}}{2 \cdot 2}$$

$$X = \frac{-7 \pm \sqrt{49-32}}{4} = \frac{-7 \pm \sqrt{17}}{4}$$

$$X = \frac{-7 \pm \sqrt{17}}{4}$$

$$X = \frac{-7 + \sqrt{17}}{4} \text{ or } \frac{-7 - \sqrt{17}}{4}$$

Practice Problems 1

Solve for X. Try factoring first, and then use the quadratic formula if necessary.

1. $X^2 - 25 = 0$
2. $X^2 - 18X = -81$
3. $2X^2 + 7X + 6 = 0$
4. $3X^2 + X - 4 = 0$
5. $4A^2 - 36 = 0$
6. $X^2 + 5 = -3X$
7. $7X^2 = -2X + 1$
8. $2X^2 + 2X - 5 = 0$
9. $\dfrac{5}{X+3} + \dfrac{2}{X-3} = 5 \quad (X \neq \pm 3)$
10. $4X^2 = 9$
11. $4X^2 + 20X = -25$
12. $3Q^2 = -4Q - 2$

Solutions 1

1. $(X+5)(X-5) = 0$

   $X+5=0 \qquad X-5=0$
   $X=-5 \qquad X=5$

2. $(X-9)(X-9) = 0$

   $X-9=0 \qquad X-9=0$
   $X=9 \qquad X=9$

3. $(2X+3)(X+2) = 0$

   $2X+3=0 \qquad X+2=0$
   $2X=-3$
   $X=-3/2 \qquad X=-2$

4. $(3X+4)(X-1) = 0$

   $3X+4=0 \qquad X-1=0$
   $3X=-4$
   $X=-4/3 \qquad X=1$

5. $(2A-6)(2A+6) = 0$

   $2A-6=0 \qquad 2A+6=0$
   $2A=6 \qquad 2A=-6$
   $A=6/2 \qquad A=-6/2$
   $A=3 \qquad A=-3$

6. $X = \dfrac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1}$

$X = \dfrac{-3 \pm \sqrt{-11}}{2} = \dfrac{-3 + i\sqrt{11}}{2}$ or $X = \dfrac{-3 - i\sqrt{11}}{2}$

7. $X = \dfrac{-2 \pm \sqrt{2^2 - 4 \cdot 7 \cdot -1}}{2 \cdot 7} = \dfrac{-2 \pm 4\sqrt{2}}{14} = \dfrac{-1 + 2\sqrt{2}}{7}$ or $X = \dfrac{-1 - 2\sqrt{2}}{7}$

8. $X = \dfrac{-2 \pm \sqrt{2^2 - 4 \cdot 2 \cdot -5}}{2 \cdot 2} = \dfrac{-2 \pm 2\sqrt{11}}{4} = \dfrac{-1 + \sqrt{11}}{2}$ or $X = \dfrac{-1 - \sqrt{11}}{2}$

9. $\dfrac{5}{X+3} + \dfrac{2}{X-3} = 5$

$5\left(X-3\right) + 2\left(X+3\right) = 5\left(X^2 - 9\right)$

$7X - 9 = 5X^2 - 45$

$5X^2 - 7X - 36 = 0$

$X = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4 \cdot 5 \cdot -36}}{2 \cdot 5}$

$X = \dfrac{7 \pm \sqrt{769}}{10}$

$X = \dfrac{7 + \sqrt{769}}{10}$ or $X = \dfrac{7 - \sqrt{769}}{10}$

10. $\left(2X-3\right)\left(2X+3\right) = 0$

$2X - 3 = 0 \qquad 2X + 3 = 0$

$2X = 3 \qquad 2X = -3$

$X = 3/2 \qquad X = -3/2$

11. $\left(2X+5\right)\left(2X+5\right) = 0$

$2X + 5 = 0 \qquad 2X + 5 = 0$

$2X = -5 \qquad 2X = -5$

$X = -5/2 \qquad X = -5/2$

12. $3Q^2 + 4Q + 2 = 0$

$X = \dfrac{-(4) \pm \sqrt{(4)^2 - 4 \cdot 3 \cdot 2}}{2 \cdot 3}$

$X = \dfrac{-4 \pm \sqrt{16 - 24}}{2 \cdot 3} = \dfrac{-4 \pm \sqrt{-8}}{2 \cdot 3}$

$X = \dfrac{-4 \pm i\sqrt{2 \cdot 4}}{2 \cdot 3} = \dfrac{-\cancel{4} \pm \cancel{2}i\sqrt{2}}{\cancel{2} \cdot 3}$

$X = \dfrac{-2 + i\sqrt{2}}{3}$ or $\dfrac{-2 - i\sqrt{2}}{3}$

Student Text: Lesson Practice 12A

Find the roots, using the quadratic formula when necessary.

1. $X^2 + 6X + 2 = 0$

2. $X^2 - 5X + 4 = 0$

3. $3X^2 + 7X - 1 = 0$

4. $A^2 - 10A = 11$

5. $2Q^2 + 2 = 17Q$

Student Text: Lesson Practice 12A

6. $5X^2 + 15X + 10 = 0$

7. $1/4\ R^2 - 1/2\ R + 3/2 = 0$

8. $16X^2 = 2X + 4$

9. $2X^2 + 3X - 8 = 0$

10. $Y^2 = 3/4\ Y + 2$

Student Text: Lesson Practice 12B

Find the roots, using the quadratic formula when necessary.

1. $8X^2 - X - 3 = 0$

2. $7 = 2X^2 + X$

3. $Q^2 - 6Q + 3 = 0$

4. $2 + 3X + 4X^2 = 0$

5. $P = P^2 - 2$

Student Text: Lesson Practice 12B

6. $X^2 + 1/5\ X + 5 = 0$

7. $20X^2 + 40X = 30$

8. $5A^2 + 2A - 1 = 0$

9. $3X^2 = -5X$

10. $AX^2 + BX + C = 0$

Student Text: Systematic Review 12C

Find the roots, using the quadratic formula when necessary.

1. $X^2 - 5X + 6 = 0$

2. $X^2 + 4X + 2 = 0$

3. $X^2 - 3X + 1 = -6X$

4. $X^2 + 4X - 12 = 0$

5. $2X^2 + 2X + 5 = 0$

6. $X^2 + 8X = -16$

Complete the square.

7. $X^2 - 26X +$ _____

8. $2X^2 + 9X +$ _____

9. $X^2 +$ _____ $+ 400$

10. $X^2 -$ _____ $+ 14$

Solve for X. Complete the square when necessary.

11. $X^2 + 1/3\,X - 4/3 = 0$

12. Check the answers to #11 by placing them in the original equation.

Student Text: Systematic Review 12C

13. Expand $(X - A)^6$.

14. What is the second term of $(1/2\ X - 3A)^4$?

15. Expand $(5 - 2A)^3$.

16. Find the cube root of $X^3 - 6X^2Y + 12XY^2 - 8Y^3$.

Put in standard form.

17. $\dfrac{6+5i}{3i-2}$

18. $\dfrac{2+\sqrt{-49}}{2-\sqrt{-49}}$

Simplify, and combine like terms when possible.

19. $\dfrac{2}{3-\sqrt{7}}$

20. $\dfrac{2+\sqrt{5}}{2\sqrt{5}-4}$

Student Text: Systematic Review 12D

Find the roots, using the quadratic formula when necessary.

1. $2X^2 - 9X - 7 = 0$

2. $X^2 + 5X - 2 = 0$

3. $3X^2 + 7X + 4 = 0$

4. $X^2 - 6X + 12 = 0$

5. $5X^2 - 3X - 2 = 0$

6. $4X^2 + 1 = 4X$

Complete the square.

7. $X^2 + 5X +$ ____

8. $X^2 - 1/2\ X +$ ____

9. $25X^2 +$ ____ $+ 1$

10. $49X^2 -$ ____ $+ 4$

Solve for X. Complete the square when necessary.

11. $X^2 - 12X + 20 = 0$

12. Check the answers to #11 by placing them in the original equation.

Student Text: Systematic Review 12D

13. Expand $(X + 1)^4$.

14. What is the fifth term of $(1/2\ X - 3A)^4$?

15. Expand $(10 - 1/X)^3$.

16. Find the cube root of $X^3 + 6X^2 + 12X + 8$.

Put in standard form.

17. $\dfrac{4 - 3i}{2i}$

18. $\dfrac{10 + \sqrt{-A}}{10 - \sqrt{-A}}$

Simplify, and combine like terms when possible.

19. $\dfrac{9}{7 + \sqrt{10}}$

20. $\dfrac{4 - \sqrt{6}}{3\sqrt{7} + 5}$

Test Booklet: Lesson 12 Test

Circle your answer.

1. Which of the following cannot be solved using the quadratic equation?

   A. $X^2 - 64 = 0$
   B. $X^3 + 3Y + 1 = 0$
   C. $4A^2 + 8A = 16$
   D. $Y^2 = 2Y + 4$

2. The part of the quadratic formula written under the radical is:

   A. $B^2 + 4AC$
   B. $B^2 - 4AC$
   C. $-B^2 \pm 4AC$
   D. $A^2 + 4BC$

3. All quadratic equations can be solved by:

   A. factoring
   B. both factoring and the quadratic formula
   C. the quadratic formula
   D. none of the above

4. In order to find values of A, B, and C in the quadratic formula, an equation should be in the form:

   A. $AX^2 = BX + C$
   B. $X^2 + AX = B - C$
   C. $AX^2 + BX + C = 0$
   D. $AX^2 + BX = -C$

5. The solution to $7X^2 + 2X - 1 = 0$ can be written as:

   A. $X = \frac{-2 \pm \sqrt{2^2 - (4)(7)(-1)}}{2(7)}$
   B. $X = \frac{2 \pm \sqrt{2^2 - (4)(7)(-1)}}{2(7)}$
   C. $X = \frac{-2 \pm \sqrt{2^2 + (4)(7)(-1)}}{2(7)}$
   D. $X = \frac{-2 \pm \sqrt{(-2)^2 - (4)(7)(-1)}}{2}$

For #6–10, solve using the best method.

6. $X^2 - 36 = 0$

   A. $X = 6, -6$
   B. $X = 4, 9$
   C. $X = 0, 6$
   D. $X = \pm 9$

7. $X^2 + 3 = -3X$

   A. $X = \frac{-3 \pm \sqrt{3}}{2}$
   B. $X = \frac{-3 \pm i\sqrt{3}}{6}$
   C. $X = \frac{3 \pm i\sqrt{3}}{2}$
   D. $X = \frac{-3 \pm i\sqrt{3}}{2}$

Test Booklet: Lesson 12 Test

8. $5X^2 = -2X + 1$

   A. $X = \frac{-1 \pm \sqrt{5}}{5}$

   B. $X = \frac{-1 \pm \sqrt{6}}{5}$

   C. $X = \frac{1 \pm 2\sqrt{6}}{5}$

   D. $X = \frac{1 \pm \sqrt{5}}{5}$

9. $4X^2 + 20X = -25$

   A. $X = \pm\ 5/2$
   B. $X = 4, 5$
   C. $X = 5/2$
   D. $X = -5/2$

10. $4X^2 + 4X - 10 = 0$

   A. $X = \frac{-1 \pm i\sqrt{11}}{2}$

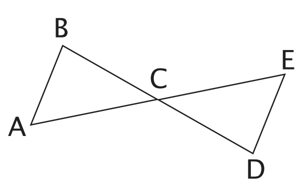
   B. $X = i, -2i$

   C. $X = \frac{-1 \pm \sqrt{11}}{2}$

   D. $X = \frac{-1 \pm 3i}{2}$

11. ΔABC is congruent to ΔEDC. $\overline{AB}$ corresponds to:

   A. $\overline{BA}$
   B. $\overline{AC}$
   C. $\overline{ED}$
   D. $\overline{BC}$



12. A quadrilateral with only one pair of parallel sides is a:

   A. rhombus
   B. trapezoid
   C. parallelogram
   D. regular polygon

13. Two sides of triangle A are congruent to the corresponding sides of triangle B. The angle formed by the corresponding sides is 25º in both triangles. What postulate may be used to prove triangles A and B congruent?

   A. SSS
   B. SSA
   C. SAS
   D. cannot be proved congruent

14. Each angle of triangle ABC is congruent to the corresponding angle of triangle DEF. What postulate may be used to prove ΔABC and ΔDEF congruent?

   A. SSS
   B. AAA
   C. SAS
   D. cannot be proved congruent

15. Five yards are a little less than:

   A. 5 meters
   B. 10 meters
   C. 2 meters
   D. 6 meters

12A

1) $\dfrac{-(6) \pm\sqrt{(6)^2 - 4(1)(2)}}{2(1)} =$

$\dfrac{-6 \pm\sqrt{28}}{2} = \dfrac{-6 \pm 2\sqrt{7}}{2} = -3 \pm\sqrt{7}$

2) $(X - 4)(X - 1) = 0$

$X - 4 = 0 \qquad X - 1 = 0$

$X = 4 \qquad X = 1$

3) $\dfrac{-(7) \pm\sqrt{(7)^2 - 4(3)(-1)}}{2(3)} = \dfrac{-7 \pm\sqrt{61}}{6}$

4) $A^2 - 10A - 11 = 0$

$(A - 11)(A + 1) = 0$

$A - 11 = 0 \qquad A + 1 = 0$

$A = 11 \qquad A = -1$

5) $2Q^2 - 17Q + 2 = 0$

$\dfrac{-(-17) \pm\sqrt{(-17)^2 - 4(2)(2)}}{2(2)} = \dfrac{17 \pm\sqrt{273}}{4}$

6) $X^2 + 3X + 2 = 0$

$(X + 1)(X + 2) = 0$

$X + 1 = 0 \qquad X + 2 = 0$

$X = -1 \qquad X = -2$

7) $R^2 - 2R + 6 = 0$

$\dfrac{-(-2) \pm\sqrt{(-2)^2 - 4(1)(6)}}{2(1)} =$

$\dfrac{2 \pm\sqrt{-20}}{2} = \dfrac{2 \pm 2i\sqrt{5}}{2} = 1 \pm i\sqrt{5}$

8) $8X^2 - X - 2 = 0$

$\dfrac{-(-1) \pm\sqrt{(-1)^2 - 4(8)(-2)}}{2(8)} = \dfrac{1 \pm\sqrt{65}}{16}$

9) $\dfrac{-(3) \pm\sqrt{(3)^2 - 4(2)(-8)}}{2(2)} = \dfrac{-3 \pm\sqrt{73}}{4}$

10) $4Y^2 - 3Y - 8 = 0$

$\dfrac{-(-3) \pm\sqrt{(-3)^2 - 4(4)(-8)}}{2(4)} = \dfrac{3 \pm\sqrt{137}}{8}$

12B

1) $\dfrac{-(-1) \pm\sqrt{(-1)^2 - 4(8)(-3)}}{2(8)} = \dfrac{1 \pm\sqrt{97}}{16}$

2) $2X^2 + X - 7 = 0$

$\dfrac{-(1) \pm\sqrt{(1)^2 - 4(2)(-7)}}{2(2)} = \dfrac{-1 \pm\sqrt{57}}{4}$

3) $\dfrac{-(-6) \pm\sqrt{(-6)^2 - 4(1)(3)}}{2(1)} = \dfrac{6 \pm\sqrt{24}}{2} =$

$\dfrac{6 \pm 2\sqrt{6}}{2} = 3 \pm\sqrt{6}$

4) $4X^2 + 3X + 2 = 0$

$\dfrac{-(3) \pm\sqrt{(3)^2 - 4(4)(2)}}{2(4)} = \dfrac{-3 \pm\sqrt{-23}}{8} =$

$\dfrac{-3 \pm i\sqrt{23}}{8}$

5) $P^2 - P - 2 = 0$

$(P - 2)(P + 1) = 0$

$P - 2 = 0 \qquad P + 1 = 0$

$P = 2 \qquad P = -1$

6) $5X^2 + X + 25 = 0$

$\dfrac{-(1) \pm\sqrt{(1)^2 - 4(5)(25)}}{2(5)} = \dfrac{-1 \pm i\sqrt{499}}{10}$

7) $2X^2 + 4X - 3 = 0$

$\dfrac{-(4) \pm\sqrt{(4)^2 - 4(2)(-3)}}{2(2)} =$

$\dfrac{-4 \pm\sqrt{40}}{4} = \dfrac{-4 \pm 2\sqrt{10}}{4} = \dfrac{-2 \pm\sqrt{10}}{2}$

8) $\dfrac{-(2) \pm\sqrt{(2)^2 - 4(5)(-1)}}{2(5)} = \dfrac{-2 \pm\sqrt{24}}{10} =$

$\dfrac{-2 \pm 2\sqrt{6}}{10} = \dfrac{-1 \pm\sqrt{6}}{5}$

9) $3X^2 + 5X = 0$

$X(3X + 5) = 0$

$X = 0 \qquad 3X + 5 = 0$

$X = 0, -5/3$

10) $\dfrac{-B \pm\sqrt{B^2 - 4AC}}{2A}$

12C

1) $(X - 3)(X - 2) = 0$
$X = 3 \quad X = 2$

2) $\frac{-4 \pm \sqrt{4^2 - 4 \cdot 1 \cdot 2}}{2 \cdot 1} = \frac{-4 \pm 2\sqrt{2}}{2} = -2 \pm \sqrt{2}$

3) $X^2 + 3X + 1 = 0$
$\frac{-3 \pm \sqrt{3^2 - 4 \cdot 1 \cdot 1}}{2 \cdot 1} = \frac{-3 \pm \sqrt{5}}{2}$

4) $(X + 6)(X - 2) = 0$
$X = -6 \quad X = 2$

5) $\frac{-2 \pm \sqrt{2^2 - 4 \cdot 2 \cdot 5}}{2 \cdot 2} = \frac{-2 \pm \sqrt{-36}}{4} =$
$\frac{-2 \pm 6i}{4} = \frac{-1 \pm 3i}{2}$

6) $X^2 + 8X + 16 = 0$
$(X + 4)(X + 4) = 0 \quad X = -4$

7) 169

8) $2X^2 + 9X +$ ____
$X^2 + 9/2\ X +$ ____
$X^2 + 9/2\ X + 81/16$
$2X^2 + 9X + 81/8$

9) $40X$

10) $2\sqrt{14}\ X$

11) $X^2 + 1/3\ X - 4/3 = 0$
$X^2 + 1/3\ X + 1/36 = 4/3 + 1/36$
$(X + 1/6)^2 = 48/36 + 1/36$
$\sqrt{(X + 1/6)^2} = \sqrt{49/36}$
$X + 1/6 = \pm 7/6$
$X = -1/6 + 7/6 \quad X = -1/6 - 7/6$
$X = 6/6 = 1 \quad X = -8/6 = -4/3$

12) $1^2 + 1/3(1) - 4/3 = 0$
$4/3 - 4/3 = 0$
$(-4/3)^2 + 1/3(-4/3) - 4/3 = 0$
$16/9 - 4/9 - 12/9 = 0$

13) $X^6 + 6X^5(-A) + 15X^4(-A)^2 + 20X^3(-A)^3 +$
$15X^2(-A)^4 + 6X(-A)^5 + (-A)^6 =$
$X^6 - 6X^5A + 15X^4A^2 - 20X^3A^3 +$
$15X^2A^4 - 6XA^5 + A^6$

14) $4/1(1/2X)^3(-3A)^1 = 4(1/8X)^3(-3A) =$
$-3/2\ X^3A$

15) $5^3 + 3(5)^2(-2A) + 3(5)(-2A)^2 + (-2A)^3 =$
$125 - 150A + 60A^2 - 8A^3$

16) $(X - 2Y)^3$

17) $\frac{(6 + 5i)\ (3i + 2)}{(3i - 2)\ (3i + 2)} =$
$\frac{18i + 15i^2 + 12 + 10i}{9i^2 - 4} = \frac{28i - 3}{-13}$

18) $\frac{(2 + \sqrt{-49})(2 + \sqrt{-49})}{(2 - \sqrt{-49})(2 + \sqrt{-49})}$
$\frac{4 + 4\sqrt{49} - 49}{4 - (-49)} = \frac{-45 + 4 \cdot 7i}{53} =$
$\frac{-45 + 28i}{53}$

19) $\frac{2(3 + \sqrt{7})}{(3 - \sqrt{7})(3 + \sqrt{7})} = \frac{6 + 2\sqrt{7}}{9 - 7} =$
$\frac{6 + 2\sqrt{7}}{2} = 3 + \sqrt{7}$

20) $\frac{(2 + \sqrt{5})(2\sqrt{5} + 4)}{(2\sqrt{5} - 4)(2\sqrt{5} + 4)} =$
$\frac{4\sqrt{5} + 8 + 2 \cdot 5 + 4\sqrt{5}}{4 \cdot 5 - 16} =$
$\frac{8\sqrt{5} + 18}{4} = \frac{4\sqrt{5} + 9}{2}$

12D

1) $\frac{-(-9) \pm \sqrt{(-9)^2 - 4(2)(-7)}}{2(2)} = \frac{9 \pm \sqrt{137}}{4}$

2) $\frac{-5 \pm \sqrt{5^2 - 4(1)(-2)}}{2} = \frac{-5 \pm \sqrt{33}}{2}$

3) $(3X + 4)(X + 1) = 0$
$X = -4/3,\ -1$

4) $\frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(12)}}{2(1)} = \frac{6 \pm \sqrt{-12}}{2} =$
$\frac{6 \pm 2i\sqrt{3}}{2} = 3 \pm i\sqrt{3}$

5) $(5X + 2)(X - 1) = 0$
$X = -2/5,\ 1$

6) $4X^2 - 4X + 1 = 0$
$(2X - 1)(2X - 1) = 0 \quad X = 1/2$

7) 25/4

8) 1/16

9) $25X^2 +$ ___ $+ 1$
$X^2 +$ ___ $+ 1/25$
$X^2 + 2/5\ X + 1/25$
$25X^2 + 10X + 1$

10) $49X^2 -$ ___ $+ 4$
$X^2 -$ ___ $+ 4/49$
$X^2 - 4/7\ X + 4/49$
$49X^2 - 28X + 4$

11) $(X - 10)(X - 2) = 0$
$X = 10,\ 2$

12) $(10)^2 - 12(10) + 20 = 0$
$100 - 120 + 20 = 0$
$(2)^2 - 12(2) + 20 = 0$
$4 - 24 + 20 = 0$

13) $X^4 + 4X^3 + 6X^2 + 4X + 1$

14) $\frac{4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 2 \cdot 3 \cdot 4}\ (1/2\ X)^0(3A)^4 = 81A^4$

15) $10^3 + 3(10)^2(-1/X) + 3(10)(-1/X)^2 + (-1/X)^3$
$1000 - 300/X + 30/X^2 - 1/X^3$

16) $(X + 2)^3$

17) $\frac{(4 - 3i)(i)}{(2i)(i)} = \frac{4i - 3i^2}{2i^2} =$
$\frac{4i + 3}{-2}$

18) $\frac{(10 + \sqrt{-A})(10 + \sqrt{-A})}{(10 - \sqrt{-A})(10 + \sqrt{-A})}$
$\frac{100 + 20i\sqrt{A} - A}{100 - (-A)} =$
$\frac{100 + 20i\sqrt{A} - A}{100 + A}$

19) $\frac{(9)\quad (7 - \sqrt{10})}{(7 + \sqrt{10})(7 - \sqrt{10})} = \frac{63 - 9\sqrt{10}}{49 - \sqrt{10}} =$
$\frac{63 - 9\sqrt{10}}{39} = \frac{21 - 3\sqrt{10}}{13}$

20) $\frac{(4 - \sqrt{6})(3\sqrt{7} - 5)}{(3\sqrt{7} + 5)(3\sqrt{7} - 5)} =$
$\frac{12\sqrt{7} - 20 - 3\sqrt{42} + 5\sqrt{6}}{9(7) - 25} =$
$\frac{12\sqrt{7} - 20 - 3\sqrt{42} + 5\sqrt{6}}{38}$

12E

1) $(X + 4)(X - 2) = 0$
$X = -4 \qquad X = 2$

2) $X - 6X + 8 = 0$
$(X - 4(X - 2) = 0$
$X = 4,\ 2$

3) $(2X - 1)(X - 7) = 0$
$X = 1/2,\ 7$

4) $3X^2 + 4X - 7 = 0$
$(3X + 7)(X - 1) = 0$
$3X + 7 = 0; \quad X - 1 = 0$
$3X = -7 \qquad X = 1$
$X = -7/3$

5) $X^2 + 5X - 2 = 0$
$\frac{-5 \pm \sqrt{5^2 - 4(1)(-2)}}{2(1)} = \frac{-5 \pm \sqrt{33}}{2}$

6) $(X + 5)(X - 3) = 0$
$X = -5,\ 3$

7) $\frac{4X^2}{4} + \frac{28X}{4} + \_\_\_\_$
$X^2 + 7X + \frac{49}{4}$
You might want to leave it in this reduced form if you were going on to solve the equation.
or, $4X^2 + 28X + 49$

8) $\frac{9X^2}{9} - \frac{36X}{9} + \_\_\_\_$
$X^2 - 4X + 4$
or, $9X^2 - 36X + 36$

9) $60X$

10) $198X$

11) $(X + 7)(X - 2) = 0$
$X = -7,\ 2$

12) $(-7)^2 + 5(-7) - 14 = 0$
$49 - 35 - 14 = 0$
$(2)^2 + 5(2) - 14 = 0$
$4 + 10 - 14 = 0$

13) $(2X)^5 + 5(2X)^4 + 10(2X)^3 + 10(2X)^2 + 5(2X) + 1$
$32X^5 + 80X^4 + 80X^3 + 40X^2 + 10X + 1$

14) $\frac{5 \cdot 4}{1 \cdot 2}\ (1/3\ X)^3(2)^2 = 10(1/27)X^3(4) =$
$40/27\ X^3$

15) $X^3 + 3X^2(-3/5) + 3X(-3/5)^2 + (-3/5)^3 =$
$X^3 - 9/5\ X^2 + 27/25\ X - 27/125$

16) $(2X + 1)^3$

17) $\frac{(10 + i)i}{(5i)i} = \frac{10i - 1}{-5}$

18) $\frac{(10)(5 + \sqrt{8})}{(5 - \sqrt{8})(5 + \sqrt{8})}$
$\frac{50 + 10\sqrt{8}}{25 - 8} = \frac{50 + 10\sqrt{4}\sqrt{2}}{17} =$
$\frac{50 + 20\sqrt{2}}{17}$

19) $\frac{(2 + 3\sqrt{6})(1 + \sqrt{6})}{(1 - \sqrt{6})(1 + \sqrt{6})} =$
$\frac{2 + 2\sqrt{6} + 3\sqrt{6} + 18}{1 - 6} =$
$\frac{20 + 5\sqrt{6}}{-5} = -4 - \sqrt{6}$

20) $\frac{(6 - \sqrt{2})(10\sqrt{3} + 8)}{(10\sqrt{3} - 8)(10\sqrt{3} + 8)} =$
$\frac{60\sqrt{3} + 48 - 10\sqrt{6} - 8\sqrt{2}}{100(3) - 64} =$
$\frac{30\sqrt{3} + 24 - 5\sqrt{6} - 4\sqrt{2}}{118}$

13A

1) $6^2 - 4(1)(9) = 0$
real, rational, equal (double root)
$(X + 3)(X + 3) = 0$
$X = -3$

2) $7^2 - 4(2)(3) = 25$
real, rational, unequal
$(2X + 1)(X + 3) = 0$
$2X + 1 = 0 \qquad X + 3 = 0$
$X = -1/2 \qquad X = -3$

3) $-2X^2 + 3X + 6 = 0$
$(3)^2 - 4(-2)(6) = 57$
real, irrational, unequal
$\frac{-3 \pm \sqrt{57}}{2(-2)} = \frac{-3 \pm \sqrt{57}}{-4}$

4) $(-2)^2 - 4(3)(5) = -56$
imaginary
$\frac{-(-2) \pm \sqrt{-56}}{2(3)} = \frac{2 \pm 2i\sqrt{14}}{6} = \frac{1 \pm i\sqrt{14}}{3}$

5) $7X^2 - 3X - 20 = 0$
$(-3)^2 - 4(7)(-20) = 569$
real, irrational, unequal
$\frac{-(-3) \pm \sqrt{569}}{2(7)} = \frac{3 \pm \sqrt{569}}{14}$

13B

1) $2R^2 + 5R - 3 = 0$
$5^2 - 4(2)(-3) = 49$
real, rational, unequal
$(2R - 1)(R + 3) = 0$
$2R - 1 = 0 \qquad R + 3 = 0$
$R = 1/2 \qquad R = -3$

2) $X^2 + 8X + 16 = 0$
$8^2 - 4(1)(16) = 0$
real, rational, equal (double roots)
$(X + 4)(X + 4) = 0$
$X = -4$

3) $6Y^2 + 7Y + 11 = 0$
$7^2 - 4(6)(11) = -215$
imaginary
$\frac{-7 \pm \sqrt{-215}}{2(6)} = \frac{-7 \pm i\sqrt{215}}{12}$

4) $4X^2 + 5X + 1 = 0$
$5^2 - 4(4)(1) = 9$
real, rational, unequal
$(4X + 1)(X + 1) = 0$
$4X + 1 = 0 \qquad X + 1 = 0$
$X = -1/4 \qquad X = -1$

5) $(-5)^2 - 4(6)(-3) = 97$
real, irrational, unequal
$\frac{-(-5) \pm \sqrt{97}}{2(6)} = \frac{5 \pm \sqrt{97}}{12}$

Test 12

1) B A, C and D are equations whose highest exponent is equal to 2. The quadratic equation works for equations of this nature only.

2) B $\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

3) C All quadratic equations have constants A, B and C which can be substituted into the quadratic formula. Not every quadratic equation can be factored.

4) C Standard form provides A, B and C with the proper sign.

5) A

6) A $X^2 - 36 = 0$
$(X + 6)(X - 6) = 0$
$X = 6, -6$

7) D $X^2 + 3X + 3 = 0$ A = 1, B = 3, C = 3
$\frac{-3 \pm \sqrt{9 - 4(1)(3)}}{2(1)} = \frac{-3 \pm \sqrt{-3}}{2} = \frac{-3 \pm \sqrt{3}\,i}{2}$

8) B $5X^2 + 2X - 1 = 0$ A = 5, B = 2, C = -1
$\frac{-2 \pm \sqrt{4 - 4(5)(-1)}}{2(5)} = \frac{-2 \pm \sqrt{24}}{10} =$
$\frac{-2 \pm 2\sqrt{6}}{10} = \frac{-1 \pm \sqrt{6}}{5}$

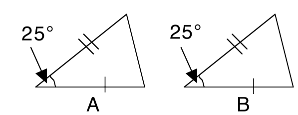
9) D $4X^2 + 20X + 25 = 0$
$(2X + 5)(2X + 5) = 0$
$X = \frac{-5}{2}$

10) C $4X^2 + 4X - 10 = 0$ A = 4, B = 4, C = -10
$\frac{-4 \pm \sqrt{16 - 4(4)(-10)}}{2(4)} = \frac{-4 \pm \sqrt{176}}{8} =$
$\frac{-4 \pm 4\sqrt{11}}{8} = \frac{-1 \pm \sqrt{11}}{2}$

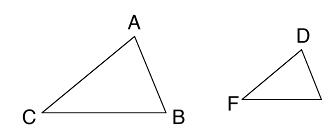
11) C

12) B A rhombus and a parallelogram have 2 pairs of parallel sides. A regular polygon may have any number of sides.

13) C SAS stands for side-angle-side



14) D Knowing that the angles of one triangle are the same as the angles of another triangle, proves similarity, not congruence.



15) A 5 m = 500 cm
5 m = 5 000 mm
5 m = .005 km

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Honors Booklet (Extra Practice): Lesson 12

You have used the binomial theorem to find the terms when a binomial is raised to a power. Here is another method that uses factorials. It was discovered by a man named Leonard Euler in the 18^{th} century.

It is based on the version of Pascal's triangle shown below. If you remember that 0! equals one, you can reduce each fraction so that this becomes a regular Pascal's triangle. Also notice that this is similar to the triangle shown in Lesson 10 in your *Teacher Manual*.

The notation $\binom{n}{r-1}$ is used in this new formula for terms of an expanded binomial. It is read as "n choose r -1".

$$\frac{0!}{0!\,0!}$$
$$\frac{1!}{0!\,1!} \quad \frac{1!}{1!\,0!}$$
$$\frac{2!}{0!\,2!} \quad \frac{2!}{1!\,1!} \quad \frac{2!}{2!\,0!}$$
$$\frac{3!}{0!\,3!} \quad \frac{3!}{1!\,2!} \quad \frac{3!}{2!\,1!} \quad \frac{3!}{3!\,0!}$$

The formula for the r^{th} term of $(a + b)^n$ is $\binom{n}{r-1}\ a^{n-r+1}\ b^{r-1}$. "n" tells you what row of the triangle you are on and "r" tells what term in that row is chosen. Remember that we start counting rows with 0. We can start counting terms with one because the formula has already subtracted one from "r".

This is not as difficult as it looks! Study the examples and compare what you are doing here to the method you have already learned.

Example 1: Find the 2^{nd} term of $(a + b)^3$. "n" = 3 and "r" = 2

$\binom{n}{r-1}\ a^{n-r+1}\ b^{r-1} = \binom{3}{2-1}\ a^{3-2+1}\ b^{2-1} = \binom{3}{1}\ a^2\ b^1$ Simplify terms

$\frac{3!}{1!\,2!}\ a^2\ b^1$ Change "n" to "n!" In this case, 3 to 3! Look at the 2^{nd} term (counting from 1) of row 3 (counting from 0) to find the factorials for the denominator.

$\frac{3\cdot\cancel{2!}}{1\cdot\cancel{2!}}\ a^2\ b^1 = 3a^2\ b$ Simplify.

Notice that the numbers in the factorial form of the denominator are the b and a exponents, and that they add to the number in the numerator. This is always true! If you remember this, you do not need to use the triangle.

Example 2: Find the 3^{rd} term of $(a + b)^6$. "n" = 6 and "r" = 3

$\binom{n}{r-1}\ a^{n-r+1}\ b^{r-1} = \binom{6}{3-1}\ a^{6-3+1}\ b^{3-1} = \binom{6}{2}\ a^4\ b^2$ Simplify terms

$\frac{6!}{2!\,4!}\ a^4\ b^2 = \frac{6\cdot 5\cdot\cancel{4!}}{2\cdot\cancel{4!}}\ a^4\ b^2 = 15a^4\ b^2$ Change to factorials and simplify.

Use factorials to find the requested term.

1) Find the 5^{th} term of $(X + Y)^6$.

2) Find the 2^{nd} term of $(A + 2)^4$.

3) Find the 3^{rd} term of $(P + Q)^5$.

4) Find the 4^{th} term of $(2X - 1)^7$.

Honors Booklet (Extra Practice) Solutions: Lesson 12

Lesson 12

1) $\binom{6}{5-1}x^{6-5+1}y^{5-1} = \binom{6}{4}x^2y^4$

$\frac{6!}{2!4!}x^2y^4 = \frac{6\times 5\times \cancel{4!}}{2\times \cancel{4!}} = 15x^2y^4$

2) $\binom{4}{2-1}A^{4-2+1}2^{2-1} = \binom{4}{1}A^3 2^1 =$

$\frac{4!}{3!1!}A^3 2 = \frac{4\times \cancel{3!}}{\cancel{3!}}A^3 2 = 4A^3 2 = 8A^3$

3) $\binom{5}{3-1}P^{5-3+1}Q^{3-1} = \binom{5}{2}P^3Q^2 =$

$\frac{5!}{3!2!}P^3Q^2 = \frac{5\times 4\times \cancel{3!}}{\cancel{3!}2\times 1} = 10P^3Q^2$

4) $\binom{7}{4-1}(2X)^{7-4+1}\left(-1^{4-1}\right) = \binom{7}{3}(2X)^4\left(-1^3\right) =$

$\frac{7!}{4!3!}\left(-16X^4\right) = \frac{7\times \cancel{6}\times 5\times \cancel{4!}}{\cancel{4!}\,\cancel{3\times 2}\times 1}\left(-16X^4\right)$

$= (35)\left(-16X^4\right) = -560X^4$

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